Biology answers and solutions

1. ribosome Level: basic ★☆☆

2. hypothalamus Level: basic ★☆☆

3. red-green color blindness Level: intermediate ★★☆

4. meiosis Level:advanced ★★★ I

5. nucleic acid (DNA or RNA) Level: intermediate ★★☆

6. test cross Level: basic ★☆☆

7. metabolism Level: intermediate ★★☆

8. endosymbiosis Level:advanced ★★★ I

9. bile Level: intermediate ★★☆

10. ATP Level:advanced ★★★ I

11. 25% (or $\frac{1}{4}$) Level:advanced ★★★ I

12. acquired immunity Level:advanced ★★★ I

13. light-dependent reactions Level:advanced ★★★ I

14. carrying capacity Level:advanced ★★★ I

15. commensalism Level: basic ★☆☆

16. $2n \rightarrow n$ Level: intermediate ★★☆

17. 55 Level:advanced ★★★ I

18. diffusion Level:advanced ★★★ I

19. natural selection Level:advanced ★★★

I

20. eukaryotic cell Level: basic ★☆☆

Mathematics answers and solutions

1. By developing the right side of the equation, we get

$(x+3)(x+b) = x^2 + (3+b)x + 3b$

So,

$x^2 - ax - 21 = x^2 + (3+b)x + 3b$

$-a = 3 + b, -21 = 3b$

$\therefore b = -7, a = 4$

Level: basic ★☆☆

2. $\frac{1}{4}\ -9\ -2\frac{5}{3}\ = \frac{1}{4}\ -9\ -\left(\frac{5}{3}\right)^{\ 2}$

$= \left\{\frac{1}{4} \times \frac{1}{9} \times \frac{9}{25}\right\}\ = \frac{1}{100}\ = 1\,0^{-2} = -2$

Level: basic ★☆☆

3. $A^{-1} = \frac{1}{4\times(-2)-5\times(-2)}(-2\ \ -5\ 2\ 4) = \frac{1}{2}(-2\ \ -5\ 2\ 4) = \left(-1\ \ -\frac{5}{2}\ 1\ 2\right)$

Level: basic ★☆☆

4. $\sum_{k=1}^{10}(k+1)(k-1) = \sum_{k=1}^{10}\left(k^2-1\right) = \sum_{k=1}^{10}k^2 - \sum_{k=1}^{10}1 = \frac{10\cdot11\cdot21}{6} - 10 = 375$

Level: basic ★☆☆

5. $\frac{x^3+x-2}{x^2-1}\ = \frac{(x-1)(x^2+x+2)}{(x-1)(x+1)} = \frac{x^2+x+2}{x+1} = \frac{1+1+2}{1+1} = 2$

Level: basic ★☆☆

6. If x->0, then denominator ->0 and numerator ->0.

Since $\lim_{x\to0}(a^x+b) = 0$, then $1+b=0 \therefore b=-1$.

If $b=-1$, then $\lim_{x\to0}\frac{a^x-1}{\ln ln\,(x+1)}\ = \lim_{x\to0}\frac{x}{\ln ln\,(x+1)}\cdot\frac{a^x-1}{x} = \ln ln\ a\ = \ln ln\ 5$

∴ a=5

∴ a-b=5-(-1)=6

Level: intermediate ★★☆

7. If $f(x)=x^2+ax+b$, then $f'(x)=2x+a$

If $f(1)=1$, then $1+a+b=1$. Also if $f'(1)=3$, then $2+a=3$.

So $a=1, b=-1$ and $f(3)=11$.

Level: intermediate ★★☆

8. $f(x)=\int \frac{x-4}{\sqrt{x}-2}dx=\int \frac{(\sqrt{x}+2)(\sqrt{x}-2)}{\sqrt{x}-2}dx$

$=\int(\sqrt{x}+2)\,dx=\int\left(x^{\frac{1}{2}}+2\right)dx$

$=\frac{2}{3}x^{\frac{3}{2}}+2x+C=\frac{2}{3}x\sqrt{x}+2x+C$
$f(1)=\frac{5}{3}$, so $\frac{5}{3}=\frac{2}{3}+2+C \therefore C=-1$

Then, $f(x)=\frac{2}{3}x\sqrt{x}+2x-1$

$\therefore f(9)=\frac{2}{3}9\sqrt{9}+18-1=35$

Level: intermediate ★★☆

9. $\{(A\cup B)\cap(A^c\cup B)\}\cup A=\{(A\cap A^c)\cup B\}\cup A$ ← distributive law

$=\{\phi\cup B\}\cup A \leftarrow A\cap A^c=\phi$

$=B\cup A=\{1,3,4,5,6\}$

So, the sum of elements is $1+3+4+5+6=19$.

10. If, $\beta-\alpha=2$, then $\beta=\alpha+2$.

So, $\alpha+(\alpha+2)=-m\cdots$①

$\alpha(\alpha+2)=15\cdots$②

From ②, $\alpha^2+2\alpha-15=0$

$(\alpha-3)(\alpha+5)=0$

$\therefore \alpha=3$ or $\alpha=-5$

If $\alpha=3$, then from ① we have $m=-8$

If $\alpha=-5$, then from ① $m=8$

$\therefore m=-8$ or $m=8$

Level: intermediate ★★☆

11. $\sum_{k=1}^{99}(1+\frac{1}{k}) = \sum_{k=1}^{99}(\frac{k+1}{k})$

$= \frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \cdots + \frac{100}{99}$

$= (\frac{2}{1} \times \frac{3}{2} \times \frac{4}{3} \times \cdots \times \frac{100}{99})$

$= 1\ 00 = 1\ 0^2 = 2$

Level: advanced ★★★

12. Let the arc length of an arc of a circle be 8. Find the maximum value of the central angle.

An arc of a circle has a length L and radius r. so $l = 8 - 2r\,(0 < r < 4)$

Then, if sector area is S,

$S = \frac{1}{2}rl = \frac{1}{2}r(8-2r)$

$= -r^2 + 4r = -(r-2)^2 + 4$.

So, when $r = 2$, S has the maximum value of 4.

We also have $S = \frac{1}{2}r^2\theta$, with θ the central angle. If S=4 and r=2, then $\theta = 2$

Level: advanced ★★★

13. If $f(x) = x^3 - ax^2 + ax$, then $f'(x) = 3x^2 - 2ax + a$

If f(x) increasing function, then $f'(x) \geq 0$

So $f'(x) = 3x^2 - 2ax + a \geq 0$

If we calculate the discriminant D for $f'(x) = 0$

$\frac{D}{4} = a^2 - 3a \leq 0, a(a-3) \leq 0$

$\therefore 0 \leq a \leq 3$

Level: advanced ★★★

14. The quadric inequality with the solution of $-2 < x < 4$ and a leading coefficient of 1 is

$(x+2)(x-4) < 0 \therefore x^2 - 2x - 8 < 0 \cdots$ ①

And, because of the the opposite direction of the sign of inequality between ① and $ax^2 + bx + c > 0$, $a < 0$.

Then, if we multiply both sides of ① by a, $ax^2 - 2ax - 8a > 0$.

This should be $ax^2 + bx + c > 0$. So, $b = -2a, c = -8a \cdots$ ②

If we put ② in $ax^2 - bx + c > 0$, then $ax^2 + 2ax - 8a > 0$.

And, if we divide both sides by a , $x^2 + 2x - 8 < 0\,(\because a < 0)$

$(x - 2)\,(x + 4) < 0 \therefore -4 < x < 2$

Level: advanced ★★★

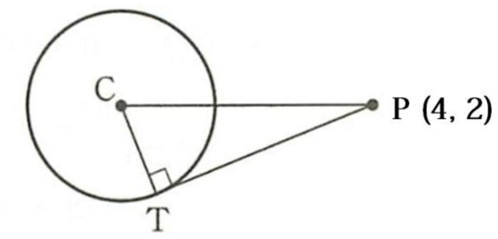
15. Let the center of a circle is C.

$\overline{CT}$ is perpendicular to $\overline{PT}$ as $\overline{PT}$ is a tangent and $\overline{CT}$ is a radius of the circle.

So triangle CTP is a right angled triangle. ($\angle CTP = 90°$)

$x^2 + y^2 + 2x + 2y - 2 = 0$, so $(x + 1)^2 + (y + 1)^2 = 4$.

The distance $P\,(4, 2)$ and $C\,(-1, -1)$,



$\sqrt{(-1-4)^2 + (-1-2)^2} = \sqrt{34}$

$\overline{CT}$ is a radius of the circle so $\overline{CT} = 2$

Using Pythagorean theorem with following right angled triangle CTP,

$\overline{PT} = \sqrt{\overline{CP}^2 - \overline{CT}^2} = \sqrt{(\sqrt{34})^2 - 2^2} = \sqrt{30}$

Level: advanced ★★★

16. Since $\sin sin\,(\frac{\pi}{2} + x)\ = \cos cos\, x$, we have $2x\ + 3\sin sin\, x\ - 3 \geq 0$.

$2x\ + 3\sin sin\, x\ - 3 \geq 0$

$2\,(1 - x\ ) + 3\sin sin\, x\ - 3 \geq 0$

$-2x\ + 3\sin sin\, x\ - 1 \geq 0$

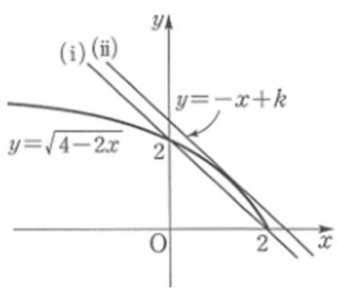
$2x\ - 3\sin sin\, x\ + 1 \leq 0$

$\therefore \frac{1}{2} \leq \sin sin\, x \leq 1$

If $0 \leq x < 2\pi$, then $\therefore \frac{\pi}{6} \leq x \leq \frac{5}{6}\pi$

Level: advanced ★★★

17. $g(x) = -x + k$ has -1 as the slope of the straight line and k as y-intersect.



(So only when $g(x) = -x + k$ lies between ① and ②, it has 2 different intersection points with $y = \sqrt{4 - 2x}$)

① is when $g(x) = -x + k$ pass through $(2, 0)$,

$0 = -2 + k \ \ \therefore k = 2$

② is when $f(x) = \sqrt{4 - 2x}$ intersects $g(x) = -x + k$ at one point, So $\sqrt{4-2x} = -x + k$.

Square each side of equation. Then $x^2 - 2(k-1)x + k^2 - 4 = 0$.

A discriminant of this equation should be 0.

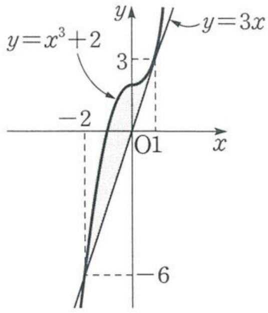
$\frac{D}{4} = (k-1)^2 - \left(k^2 - 4\right) = 0, -2k + 5 = 0 \therefore k = \frac{5}{2}$

Only when $g(x) = -x + k$ lies between ① and ②, it has 2 different intersection points with $y = \sqrt{4-2x}$

So, $2 \le k < \frac{5}{2}$

Level: advanced ★★★

18. Let $f(x) = x^3 + 2$. $f'(x) = 3x^2 \therefore f'(1) = 3$



So, equation of the tangent line at (1,3),

$y - 3 = 3(x-1) \therefore y = 3x$

The x-coordinate of the intersection point of the curve $y = x^3 + 2$ and the line $y = 3x$,

$x^3 + 2 = 3x$, $(x-1)^2(x+2) = 0$, $\therefore x = 1$ or $x = -2$

So, on the right image, if let the area of the region enclosed by the curve and the line is S,

$S = \int_{-2}^{1} \{(x^3+2) - 3x\}\, dx = \left[\frac{1}{4}x^4 - \frac{3}{2}x^2 + 2x\right]_{-2}^{1} = \frac{27}{4}$

Level: advanced ★★★

19. $f'(x) = -\sin sin\, x + \sin sin\, x + x \cdot \cos cos\, x = x \cdot \cos cos\, x$

When $f'(x) = 0$, $x = 0$ or $\cos cos\, x = 0$

$\therefore x = 0$ or $x = \frac{\pi}{2}$ or $x = \frac{3}{2}\pi$ $(\because 0 \le x \le 2\pi)$

The local maximum value: $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

The local minumum value: $f\left(\frac{3}{2}\pi\right) = -\frac{3}{2}\pi$

And $f(0) = 1, f(2\pi) = 1$

∴ Global maximum : $\frac{\pi}{2}$, Global minimum : $-\frac{3}{2}\pi$

Level: advanced ★★★

20. $\lim_{n\to\infty} \frac{1}{n^3} \left\{(3n+1)^2 + (3n+2)^2 + (3n+3)^2 + \cdots + (3n+n)^2\right\}$

$= lim_{n\to\infty} \frac{1}{n} \left\{ \frac{(3n+1)^2}{n^2} + \frac{(3n+2)^2}{n^2} + \frac{(3n+3)^2}{n^2} + \cdots + \frac{(3n+n)^2}{n^2} \right\}$

$$= lim_{n\to\infty}\frac{1}{n}\left\{\left(3+\frac{1}{n}\right)^2+\left(3+\frac{2}{n}\right)^2+\left(3+\frac{3}{n}\right)^2+\cdots+\left(3+\frac{n}{n}\right)^2\right\}$$

$$= lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(3+\frac{k}{n}\right)^2=\int_3^4 x^2dx=\left[\frac{1}{3}x^3\right]_3^4=\frac{37}{3}$$

Level: advanced ★★★

Chemistry answers and solutions

1.

Answer: the oxidizing agent : $HNO_3(aq)$, the reducing agent : $SO_2(g)$

Solution: Calculate changes in the oxidation numbers of the elements:

$3SO_2(g)$	+	$2HNO_3(aq)$	+	$2H_2O(l)$	→	$3H_2SO_4(aq)$	+	$2NO(g)$
S : +4		H : +1		H : +1		H : +1		N : +2
O : -2		N : +5		O : -2		S : +6		O : -2
		O : -2				O : -2		

The oxidation state of S increases from +4 to +6 , and that of N decreases from +5 to +2.

Level: basic ★☆☆

2.

Answer: 2, 2, 4 – trimethylhexane

Level: intermediate ★★☆

3.

Answer: mass of CO_2 remaining 440g

Solution: $moles\ of\ (NH_2)_2CO = 510g\ NH_3 \times \frac{1mol\ NH_3}{17g\ NH_3} \times \frac{1mol\ (NH_2)_2CO}{2mol\ NH_3}$

$= 15\ mol\ (NH_2)_2CO$

$moles\ of\ (NH_2)_2CO = 1100g\ CO_2 \times \frac{1mol\ CO_2}{44g\ CO_2} \times \frac{1mol\ (NH_2)_2CO}{1mol\ CO_2}$

$= 25\ mol\ (NH_2)_2CO$

Therefore, NH_3 must be the limiting reagent because it produces a smaller amount of $(NH_2)_2CO$

$mass\ of\ CO_2\ reacted = 15\ mol\ (NH_2)_2CO \times \frac{1mol\ CO_2}{1\ mol\ (NH_2)_2CO} \times \frac{44g\ CO_2}{1mol\ CO_2}$

$= 660g\ CO_2$

$mass\ of\ CO_2\ remaining = 1100\ g - 660g = 440g$

Level: intermediate ★★☆

4.

Answer: $CuSO_4 \cdot 5H_2O$

Level: basic ★☆☆

5.

Answer: $C_2H_6O_2$

Solution:

If we have 100 g of ethylene glycol, then each percentage can be converted directly to gram.

Let n represent the number of moles each element so that

$n_C = 38.7g\ C \times \frac{1mol\ C}{12g\ C} = 3.225\ mol$ C

$n_H = 9.7g\ H \times \frac{1mol\ H}{1g\ H} = 9.7\ mol$ H

$n_O = 51.6g\ O \times \frac{1mol\ O}{16g\ O} = 3.225\ mol$ O

C : H : O = $\frac{3.225}{3.225} : \frac{9.7}{3.225} : \frac{3.225}{3.225}$ = 1.00 : 3.00 : 1.00

Thus, the empirical formula is CH_3O.

Level: advanced ★★★

6.

Answer: $100mL$

Solution:

We calculate the number of moles of H_2SO_4 in a 20.0 mL solution:

$$moles\ H_2SO_4 = \frac{0.500\ mol\ \ H_2SO_4}{1000\ mL}\quad 20.0mL$$

$$= 1.00 \quad 10^{-2}\ mol\ H_2SO_4$$

From the stoichiometry we see that 1mol H_2SO_4 ≅ 2mol NaOH. Therefore, the number of moles of NaOH reacted must be 2 × 1.00 × 10^{-2} mole.

From the definition of molarity, we have

$$liters\ of\ solution = \frac{moles\ of\ solute}{molarity}$$

$$volume\ of\ NaOH = \frac{2.00 \quad 10^{-2}\ mol\ \ NaOH}{0.200\frac{mol}{L}}$$

$$= 0.1L = 100mL$$

Level: advanced ★★★

7.

Answer: 29.12L

Solution: According to Avogadro's law, at the same temperature and pressure, the number of moles of gases are related to their volumes.

From the equation we have 2mol C_4H_{10} ≅ 13mol O_2. Therefore, the volume of O_2 that will react with

2.24L C_4H_{10} is given by

$$volume\ of\ O_2 = 2.24L\ C_4H_{10} \times \frac{13L\ O_2}{2L\ C_4H_{10}}$$

$$= 29.12L$$

Level: intermediate ★★☆

8.

Answer: 10 electrons

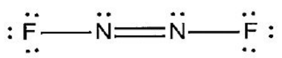
Solution: n = 3, l = 2, ml = +2, +1, 0, − 1, − 2.
Each of the five orbitals can hold 2 electrons for a total of 10 electrons
Level: advanced ★★★

9.

Answer: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^5$ or $[Ar]\ 3d^5$

Level: intermediate ★★☆

10.

Answer: 

Level: advanced ★★★

11.

Answer: 11 protons, 12 neutrons, 10 electrons

Level: basic ★☆☆

12.

Answer: 0.4M

Solution: $Molarity = \frac{moles\ solute}{volume\ of\ solution\ in\ liters}$

$$moels\ NaOH = 4.00g\ NaOH \times \frac{1mol\ NaOH}{40g\ NaOH} = 0.1\ mol\ NaOH$$

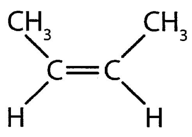
$$liters\ of\ solution = 250mL \times \frac{1L}{1000mL} = 0.250L$$

$$Molarity = \frac{0.1mol\ NaOH}{0.250L} = 0.4M$$

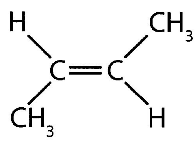
Level: intermediate ★★☆

13.

Answer:



cis-2-butene

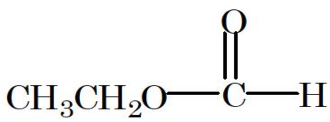


trans-2-butene

Level: advanced ★★★

14.

Answer:



Solution: Esterification reaction : R-COOH + R'-OH → RCOOR' + H_2O

Level: basic ★☆☆

15.

Answer: ΔH = -110.5 kJ

Solution: $C\,(s) \;+\; O_2\,(g) \;\rightarrow\; CO_2\,(g) \qquad \Delta H = \;-393.5\;kJ$

$CO_2\,(g) \;\rightarrow\; CO\,(g) \;+\; \frac{1}{2}O_2\,(g) \qquad -\Delta H = \;283.0\;k$

$C\,(s) \;+\; \frac{1}{2}O_2\,(g) \rightarrow CO\,(g) \qquad \Delta H = \;-110.5\;kJ$

Level: advanced ★★★

16.

Answer: concentration of reactant, temperature, activation energy, and catalyst

Level: advanced ★★★

17.

Answer: 0.06

Solution: The equilibrium constant K_P for the overall reaction is

$$K_P = \frac{P2NO}{P_{N_2}P_{O_2}} = \frac{(0.06)^2}{(0.2)(0.3)} = 0.06$$

Level: advanced ★★★

18.

Answer: He : 1.0atm, CH_4 : 0.5atm, N_2 : 1.5atm

Solution: The number of moles of the combined gases is:

$$n = n_{He} + n_{CH_4} + n_{N_2} = 0.50mol + 0.25mol + 0.75mol = 1.50mol$$

The mole fraction of each component of the mixture is:

$$X_{He} = \frac{0.50\ mol}{1.50\ mol} = 0.33 \qquad X_{CH_4} = \frac{0.25\ mol}{1.50\ mol} = 0.17 \qquad X_{N_2} = \frac{0.75\ mol}{1.50\ mol} = 0.50$$

The partial pressures are:

$$P_{He} = X_{He} \times P_{total} = 0.33 \times 3atm = 1.0atm$$

$$P_{CH_4} = X_{CH_4} \times P_{total} = 0.17 \times 3atm = 0.5atm$$

$$P_{N_2} = X_{N_2} \times P_{total} = 0.50 \times 3atm = 1.5atm$$

Level: advanced ★★★

19.

Answer: He < CH_4 < H_2O < NaCl

Solution: The attractive forces are stronger for ionic compounds than for molecular ones, so NaCl should have the highest boiling point. The boiling point of H_2 should be the lowest because it is nonpolar and has the lowest molecular weight. The molecular weights of CH_4, H_2O are similar. Because H_2O can hydrogen bond, however, it should have the highest boiling point than CH_4.

Thus, the predicted order of boiling point is He < CH_4 < H_2O < NaCl.

NaCl: ionic compound, ionic bond

He, CH_4: nonpolar molecules.

H_2O: polar molecule. Hydrogen bond.

Level: advanced ★★★

20.

Answer: $Zn^{2+} + S^{2-} \rightarrow ZnS$

Solution:

The complete ionic equation is $2Na^+ + S^{2-} + Zn^{2+} + 2Cl^- \rightarrow ZnS\downarrow + 2Na^+ + 2Cl^-$

Na^+ and Cl^- are spectator ions. Canceling them gives the following net ionic equation.

The net ionic equation: $Zn^{2+} + S^{2-} \rightarrow ZnS$

Level: basic ★☆☆